Cheat Sheet 1: Sigma Notation

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1 Removing a Factor

A simple adjustment which can help reach a situation where **2** can be applied:

$$\sum_{i=1}^{n} 3i = 3\sum_{i=1}^{n} i$$

2 Finite Arithmetic Sequences

The sum of a finite arithmetic sequence $1 + 2 + ... + n$ can be written in sigma notation as $\sum_{i=1}^{n} i$, but that can alternatively be represented as $\frac{1}{2}n(n+1)$. So:

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$$

3 Finite Geometric Sequences

$$\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r} = \frac{r^{n+1}-1}{r-1}$$

The second representation is derived by multiplying the first by -1. The former is more convenient where $-1 \leq r \leq 1$ and the latter for other values of r. Combined with **1**, we can see that:

$$\sum_{i=0}^{n} ar^i = a\left(\frac{1-r^{n+1}}{1-r}\right) = a\left(\frac{r^{n+1}-1}{r-1}\right)$$

4 Another Type of Sequence

For a sequence $x_i (i = 1, 2, ...)$ where a and b are constants:

$$\sum_{i=1}^{n} a + bx_i = an + b\sum_{i=1}^{n} x_i$$

Which can be further generalised for sequences beginning at m:

$$\sum_{i=m}^{n} a + bx_i = a(n-m+1) + b\sum_{i=m}^{n} x_i$$